Introduction to topological acoustics

Part III

14-16 December 2020

Master Wave Physics & Acoustics

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Previously

The SSH model

- Two bands
- 0 or 1 edge mode: transition at gap closing (one per boundary)

Topology

- Choice of eigen-vectors $\varphi(q)$ defines a connection $A(q)$
- A leads to accumulated phase around a loop

$$\alpha = \oint A(q)\mathrm{d}q$$

- If mirror symmetry $\Rightarrow \alpha_Z$ topological invariant

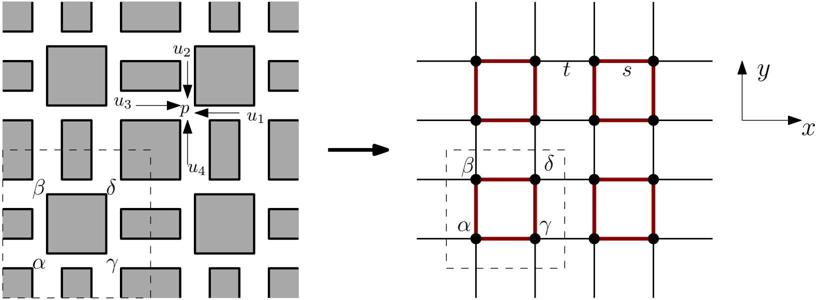
Bulk-boundary correspondance

- Topological invariant = number of edge mode

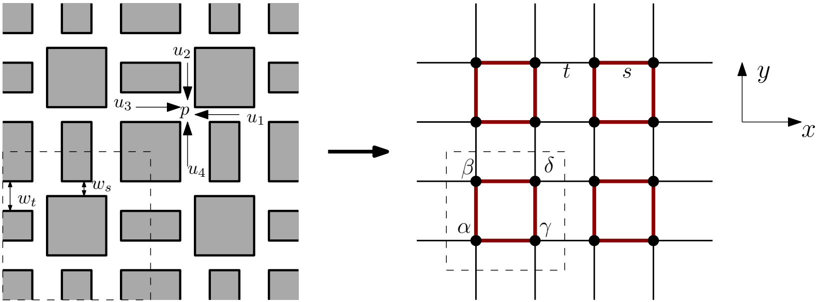
Outline

1. The 2D SSH model: infinite system
   - 2D acoustic network
   - Bloch modes
   - Topological invariants
2. The 2D SSH model: ribbon configuration

Outline



- 2D acoustic network
- Motif with 4 intersections repeats itself (dashed square)
- Consider **acoustic pressure** values at every intersections
- $\alpha_{m,n}$ is lower-left in cell number m along x and n along y
- Similarly for $\beta_{m,n}$, $\gamma_{m,n}$ and $\delta_{m,n}$

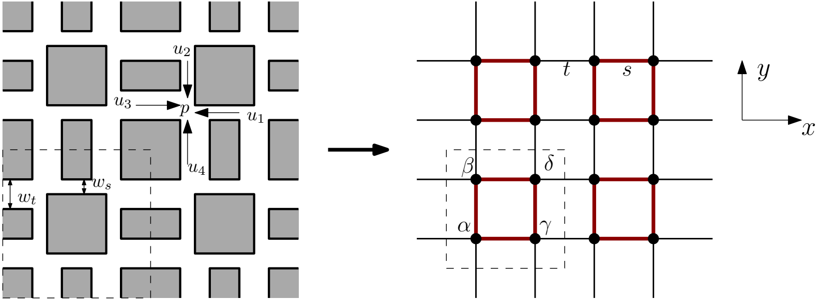


- In tubes: 1D propagation

$$\cos(kL)p + i\sin(kL)u_j = p_j,$$

- Acoustic debit conservation

$$w_s u_1 + w_s u_2 + w_t u_3 + w_t u_4 = 0.$$



- Sum over nearby intersections to eliminate u_j's

$$2(w_s+w_t)\cos(kL)\alpha_{m,n} = w_s\beta_{m,n}+w_t\beta_{m,n-1}+w_s\gamma_{m,n}+w_t\gamma_{m-1,n}$$

- 2D SSH with $\varepsilon = 2\cos(kL)$

$$s = \frac{w_s}{w_s+w_t} \qquad t = \frac{w_t}{w_s+w_t}$$

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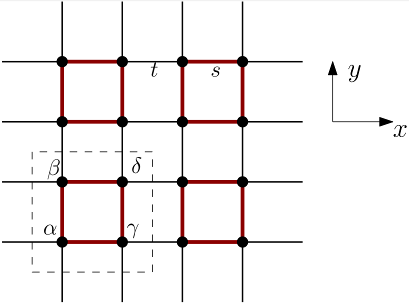
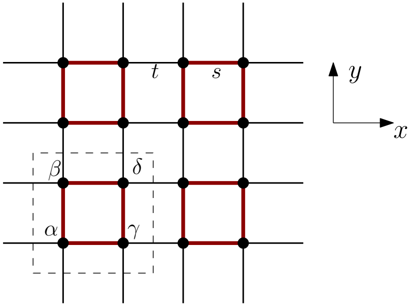


Bloch method for 2D?

- Both $\hat{T}_x$ and $\hat{T}_y$ translation operators
- This defines two wavenumbers q_x and q_y
- Each component

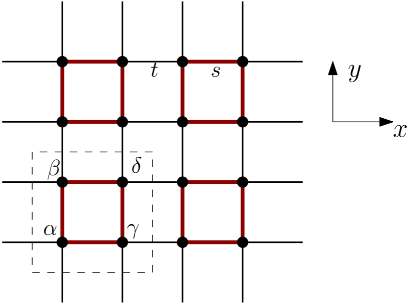
$$\alpha_{m,n} = e^{imq_x + inq_y} \alpha$$

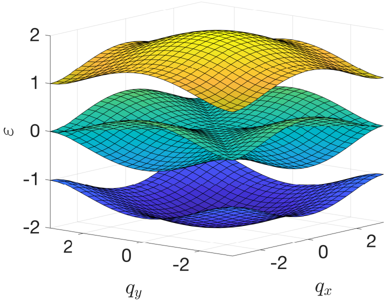
and similarly for β, γ and δ



- Bloch eigen-vectors $\varphi = \begin{pmatrix} \alpha & \beta & \gamma & \delta \end{pmatrix}^T$
- Bloch Hamiltonian

$$H(q) = \begin{pmatrix} 0 & s + te^{-iq_y} & s + te^{-iq_x} & 0 \\ s + te^{iq_y} & 0 & 0 & s + te^{-iq_x} \\ s + te^{iq_x} & 0 & 0 & s + te^{-iq_y} \\ 0 & s + te^{iq_x} & s + te^{iq_y} & 0 \end{pmatrix}$$

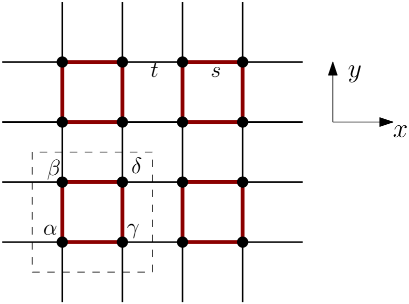


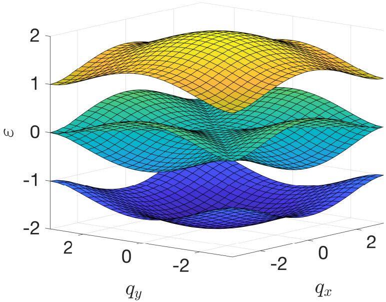


- Dispersion relation

$$\varepsilon = \varepsilon_n(q_x, q_y)$$

- $n = 1, 2, 3$ or 4

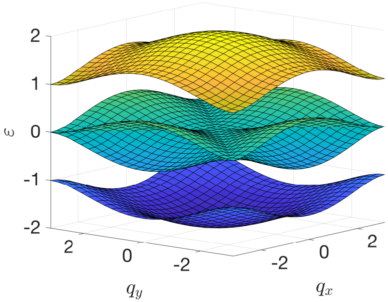




- Useful to notice: chiral symmetry again

$$\Gamma : \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix} \to \begin{pmatrix} \alpha \\ -\beta \\ -\gamma \\ \delta \end{pmatrix}$$

Outline



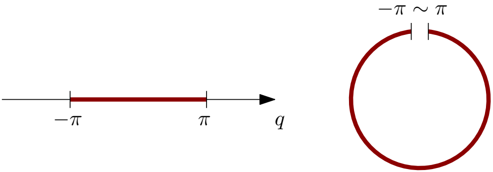
Topological invariants?

- Let's take highest band
- Build Berry connection

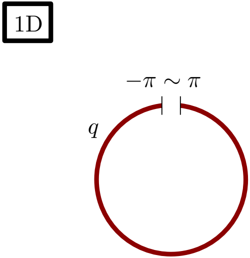
$$A(q_x, q_y) = -i\langle\varphi|\nabla\varphi\rangle$$

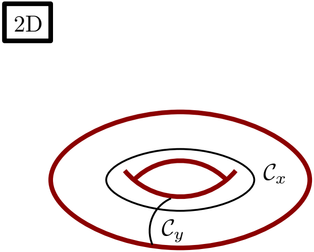
with

$$\nabla = \partial_{q_x}\mathbf{e}_1 + \partial_{q_y}\mathbf{e}_2$$



- 1D system: momentum space is a circle
- 2D system:
  - A is a vector
  - momentum space is 2D
- Which contour to compute Zak phase?





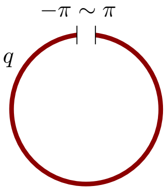
- 2D system: momentum space is a torus
- Two Zak phases

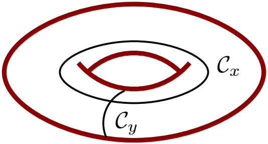
$$\alpha_x = \oint_{\mathcal{C}_x} \mathbf{A} \cdot \mathrm{d}\mathbf{q}$$

$$\alpha_y = \oint_{\mathcal{C}_y} \mathbf{A} \cdot \mathrm{d}\mathbf{q}$$

1D

2D





Computing the Zak phases

- if $s < t$:

$$\left.\begin{aligned} \alpha_x &= \pi \\ \alpha_y &= \pi \end{aligned}\right) \text{Topological}$$

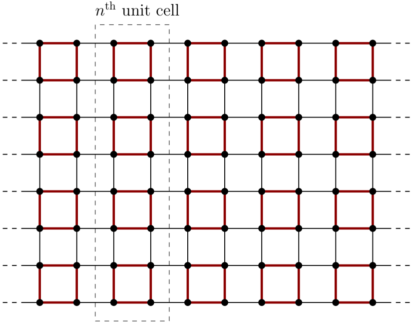
- if $t < s$:

$$\left.\begin{aligned} \alpha_x &= 0 \\ \alpha_y &= 0 \end{aligned}\right) \text{Trivial}$$

Consequences on edges?

Outline

2D SSH in a ribbon configuration



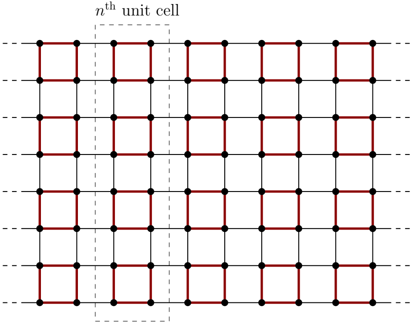
- Problem similar to acoustic in a waveguide
- Modes in a waveguide:

plane wave — transverse profile

$$p(x, y) = e^{ikx} f(y)$$

- Similar here

2D SSH in a ribbon configuration

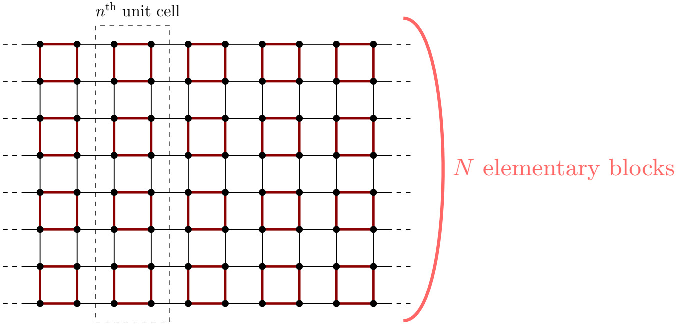


- System still periodic along x
- Modes in a 2D SSH ribbon:

$$\xi_{m,n} = e^{imq_x} \bar{\xi}_n$$

- with $\xi = \alpha$, β, γ or δ

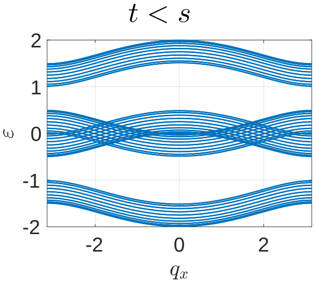
2D SSH in a ribbon configuration

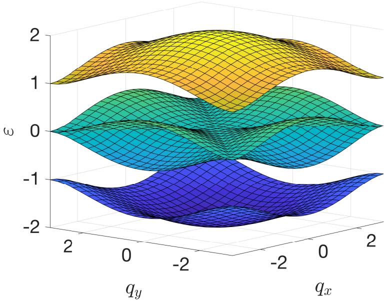


- Bloch Hamiltonian of a ribbon

$$\varepsilon(q_x)\tilde{\xi} = H_{\text{rib}}(q_x) \cdot \tilde{\xi}$$

- H_{rib} is $4N \times 4N$



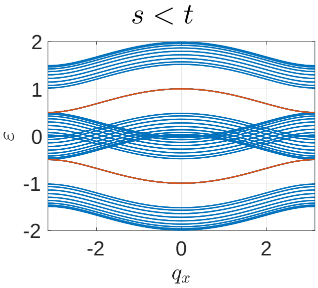


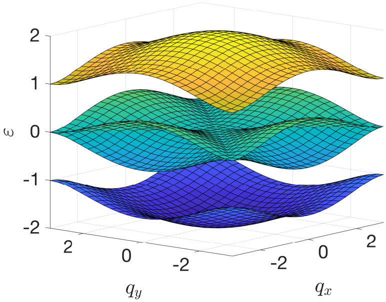
Trivial case

- Bloch Hamiltonian of a ribbon

$$\varepsilon(q_x)\tilde{\xi} = H_{\text{rib}}(q_x) \cdot \tilde{\xi}$$

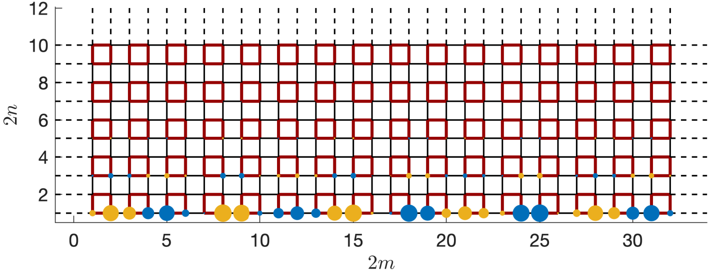
- H_{rib} is $4N \times 4N$





Topological case

- Extra mode in the gap
- **Edge** modes
  - Have their own dispersion relation
  - propagate along edges
  - Can move left or right



Topological case

- Plot of edge mode
- Localized near the edge
- Propagative

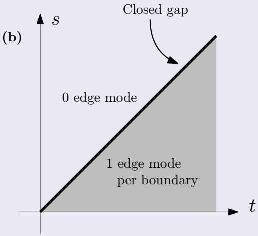
Super recap' I

Bloch-Floquet method

- Periodic systems
- Bloch condition $\varphi_q(x+a) = e^{iq}\varphi_q(x)$
- Bloch eigen-value problem $\varepsilon(q)\varphi_q = H(q)\cdot\varphi_q$
- $\varepsilon(q)$ gives energy bands ($\sim$ frequency bands)

The SSH model

- Two bands
- Edge mode if $s < t$; absence if $s > t$
- To get rid of it: **must close gap**
- **Locked** at $\varepsilon = 0$ (chiral symmetry)



Super recap' II

Bulk-boundary correspondance

$H(q)$

Infinite system: Bulk

↓

Topological invariant

$\hat{H}$

Finite system: Bulk + Boundary

↓

Edge modes

Topological invariant ⟷ Correspondance ⟷ Edge modes

Same number $n \in \mathbb{Z}$

That's all folks

Some references:

- Objective of part III: 2D SSH in acoustics
  [Zheng, Achilleos, Richoux, Theocharis, Pagneux "Observation of edge waves in a two-dimensional SSH acoustic network" (2019)]
- Reviews with treatment of SSH model:
  [Dalibard "La matière topologique et son exploration avec les gaz quantiques" (2017)]
  [Asboth, Oroszlany, Palyi "A short course on topological insulators" (2016)]
- Berry phase and topology
  [Berry "Quantum adiabatic anholonomy" (1990)]
  [Budich, Trauzettel "From the adiabatic theorem of quantum mechanics to topological states of matter" (2013)]